REGULAR GENERALIZED FUZZY B-REGULAR AND NORMAL SPACES

Varsha Joshi[1], Jenifer J. Karnel[2]

Department of Mathematics, SDMCET, Dharwad, India

ABSTRACT

This study looks at the characterization of 'regular generalized fuzzy b' - open (closed) sets.

Various kinds of "regular generalized fuzzy b'-normal space', 'regular generalized fuzzy b'-regular spaces are defined and their interrelationships investigated.

KEYWORDS: rgfb-CS, rgfb-OS, rgfbR, $rgfb_1N$, $rgfb_2N$, $rgfb_3N$, $rgfb_4N$ and fts (fuzzy topological spaces).

INTRODUCTION

Chang researched the idea of fts and Zadeh presented the basic idea of 'fuzzy sets'. Following this, Ghanimet al. presented fuzzy normal spaces, regular spaces, and separation axioms in fuzzy topology. Benchalli et al. introduced the concepts of 'fuzzy b-open sets', 'fuzzy b-generalized closed sets', 'fuzzy b-regular spaces', and 'fuzzy b-normal spaces'. Jenifer et al. introduced the idea of a 'regularized generalized fuzzy b-closed set' (open set). Jenifer et al. introduced the idea of 'regularized fuzzy b-separation axioms'. We define the 'rgfb-regular and rgfb-normal spaces' in this work, and we examine the implications. The study examines the productivity behavior of various 'fuzzy normal space' concepts.

2. PRELIMINARY

$(X_{1,\tau})$ or simple X_1 states fts.

2.1 Definition

Consider δ be a fuzzy set In fts X_1 , then

- δ is 'fuzzy regular open(precisely, frOS)' If δ =IntCl(δ)
- δ is 'fuzzy regular closed (precisely, frCS)' If δ =ClInt(δ)
- δ is 'f b-open set (precisely, fbOS)' If $\delta \leq$ (IntCl δ) V (ClInt δ)
- δ is 'f b-closed set (precisely, fbCS)' If $\delta \geq$ (IntCl δ) Λ (ClInt δ)

2.2. Remark

In a fts X_1, The inference that follows is valid.

Fig1. Relationships among a few fuzzy open sets

2.3 Definition

If bCl $(\omega) \leq \varepsilon$, at any time when $\omega \leq \varepsilon$, then is 'regular generalized fuzzy b'-closed (rgfb-CS). Where ε is frOS in X_1

2.4 Definition

If bInt $(\omega) \geq \varepsilon$, at any time when $\omega \geq \varepsilon$, then ω is 'regular generalized fuzzy b'-open (rgfb-OS).Where ε is frCS in X_1.

2.5 Remark

In a fts X_1, The inference that follows is valid.

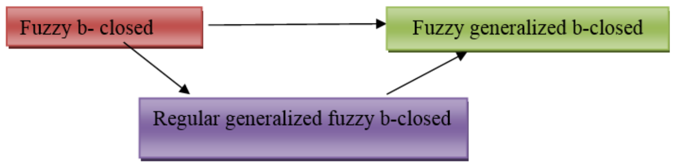

Fig 2 Relationships among a few fuzzy closed sets

2.6 Definition

Rgfb-Cl $(\omega) = \Lambda \{\varepsilon : \varepsilon$ is a rgfb-CS(X_1), $\geq \omega\}$. Where $\omega \in X_1$.
Rgfb-cl: 'regular generalized fuzzy b'-closure

2.7 Definition

Rgfb-Int(ω) = V $\{\lambda : \lambda$ is a a rgfb-OS$(X_1), \leq \omega\}$. Where $\omega \in$ X1
Rgfb-Int: 'regular generalized fuzzy b'-interior .

2.8 Definition

Let λ & μ be fuzzy sets, λ quasi-coincident with μ denoted by λ q μ iff there exist x $\in X_1$ such that $\lambda(x) + \mu(x) > 1$. If λ and μ are not quasi-coincident then $\lambda \bar{q} \mu$. $\lambda \leq \mu \leftrightarrow \lambda$ q 1- μ

2.9 Definition

A fts is known as 'regular generalized fuzzy b'T_0(rgfbT_0) iff for each pair of fuzzy singletons q_1 and q_2 with distinct supports, there exists rgfb-OS μ so that either $q_1 \leq \mu \leq 1- q_2$ or $q_2 \leq \mu \leq 1- q_1$.

2.10 Definition

A fts is known as 'regular generalized fuzzy b'$T_{2\frac{1}{2}}$(rgfb$T_{2\frac{1}{2}}$) or rgfb-Urysohn iff for each pair of fuzzy singletons q_1 and q_2 with distinct supports x_1 and x_2 respectively, there exists rgfb-OSs μ_1 and μ_1 so that $q_1 \leq \mu_1 \leq 1- q_2$, $q_2 \leq \mu_2 \leq 1- q_1$ and rgfbcl μ_1

$\leq$ 1-rgfbcl μ_2.

3. rgfb-REGULAR SPACE (rgfbR)

3.1 Definition

A fts X_1 is 'regular generalized fuzzy b'- regular space (rgfbR) if and only if for a fuzzy singleton q and a fCS λ, there exist two rgfb-OSs μ_1 and μ_2 so that, $\lambda \leq \mu_1$, $q \leq \mu_2$ and $\mu_1 \leq 1 - \mu_2$.

3.2 Theorem

A fts X_1 is rgfbR if and only if for a fuzzy singleton q and a fOS λ such that, $q \leq \lambda$, there exists a rgfb-OS μ such that, $q \leq \mu \leq$ rgfbCl $\mu \leq \lambda$.

3.4 Theorem

For rgfbR X_1, which is also rgfbT_{0}- space, it is rgfb$T_{2\frac{1}{2}}$.

Proof. Consider two fuzzy singletons p and q with different supports. Since X_1 is rgfbT_0, $p \leq U \leq$ 1-q where $U \in \tau$. Let λ= 1-U then, $p \leq 1-\lambda$. From the statement [3.3], there exist two rgfb-OS μ_1 and μ_2 such that $p \leq \mu_1$, $\lambda \leq \mu_2$ and rgfbCl $\mu_1 \leq 1 -$ rgfbCl μ_2.But $q \leq$ 1-U= λ which implies that $p \leq \mu_1$, $q \leq \mu_2$ and rgfbCl $\mu_1 \leq 1 -$ rgfbCl μ_2 Thus X_1 is rgfb$T_{2\frac{1}{2}}$.

3.5 'regular generalized fuzzy b'1 – Normal Space

A fts X_1 is 'regular generalized fuzzy b'$_1$ – normal space (precisely, rgfb$_1$N) if and only if for every pair of fCSs λ_1 and λ_2 such that, $\lambda_1 \bar{q} \lambda_2$, there exist two rgfb-OS μ_1 and μ_2 such that, $\lambda_1 \leq$, $\lambda_2 \leq \mu_2$ and $\mu_1 \wedge \mu_2$ =0.

3.6 'regular generalized fuzzy b'$_2$ – Normal Space

A fts X_1 is 'regular generalized fuzzy b'$_2$ – normal space (precisely, rgfb$_2$N) if and only if for every pair of fCSs λ_1 and λ_2 such that, $\lambda_1 \wedge \lambda_2 = 0$ there exist two rgfb-OS μ_1 and μ_2 such that, $\lambda_1 \leq \mu_1$, $\lambda_2 \leq \mu_2$ and $\mu_1 \wedge \mu_2$=0.

3.7 'regular generalized fuzzy b'$_3$ – Normal Space

A fts X_1 is 'regular generalized fuzzy b'$_3$ – normal space (precisely, rgfb$_3$N) if and only if for every pair of fCSs λ_1 and λ_2 such that, $\lambda_1 \bar{q} \lambda_2$ there exist two rgfb-OS μ_1 and μ_2 such that, $\lambda_1 \leq \mu_1$, $\lambda_2 \leq \mu_2$ and $\mu_1 \bar{q} \mu_2$.

3.8 'regular generalized fuzzy b'$_4$ – Normal Space

A fts X_1 is 'regular generalized fuzzy b'$_4$ – normal space (precisely, rgfb$_4$N) if and only if for every pair of fCSs λ_1 and λ_2 such that, $\lambda_1 \wedge \lambda_2 = 0$ there exist two rgfb-OS μ_1 and μ_1 such that, $\lambda_1 \leq \mu_1$, $\lambda_2 \leq \mu_2$ and $\mu_1 \bar{q} \mu_2$.

3.9 Proposition

In fts X_1, every rgfb$_1$ N is rgfb$_2$ N

Proof. Obvious. Since for $\lambda_1 \bar{q} \lambda_2$ the result $\lambda_1 \wedge \lambda_2 = 0$ holds good. Where λ_1 & λ_2 are fCSs.

The opposite of above statement is incorrect. This is shown as follows –

3.10 Example

Consider $X_1 = \{u,v\}$ and $\tau = \{0, O_1, O_2, O_3, O_4, O_5, O_6, O_7, 1\}$ Where, $O_1=\{(u, 0.6), (v, 1)\}$, $O_2=\{(u, 1), (v,0.6)\}$, $O_3=\{(u, 0.7), (v, 0)\}$, $O_4=\{(u, 0), (v, 0.3)\}$, $O_5=\{(u, 1), (v, 0)\}$, $O_6=\{(u, 0.6), (v, 0.6)\}$, $O_7=\{(u, 0.6), (v, 0)\}$.

A fts X_1 is rgfb$_2$ N but not rgfb$_1$ N. For fCS $\lambda_1= (u,0.4), (v,0)\}$, $\lambda_2 =\{(u, 0), (v, 1)\}$ relation $\lambda_1 \bar{q} \lambda_2$ do not holds good.

3.11 Proposition

In fts X_1, every rgfb1N is rgfb$_3$ N.

Proof. Obvious. Since for rgfb-OS μ_1 and μ_2, $\mu_1 \wedge \mu_2$ =0. and $\lambda_1 \bar{q} \lambda_2$ exists.

The opposite of above statement is incorrect. This is shown as follows –

3.12 Example

Consider $X_1=\{u,v\}$ and $\tau = \{0, O_1, O_2, O_3, O_4, O_5, O_6, 1\}$.

Where, $O_1=\{(u, 0), (v,0.6)\}$, $O_2=\{(u, 0.3), (v,0)\}$, $O_3=\{(u,1), (v,0.2)\}$, $O_4=\{(u,0.7), (v1)\}$, $O_5=\{(u,0.7), (v, 0.2)\}$, $O_6=\{(u,1), (v,0.6)\}$.

A fts X_1 is rgfb$_3$ N but not rgfb$_1$N. For rgfb-OSs O_3 and O_4 where $\lambda_1=\{(u, 0.3), (v, 0)\} \leq O_3$,

$\lambda_2 =\{(u, 0.3), (v, 0.8)\} \leq O_4$ such that, $\lambda_1 \bar{q} \lambda_2$ but $O_3 \wedge O_4 \neq 0$.

3.13 Proposition

In fts X_1, every rgfb$_2$ N is not rgfb$_3$ N and vice versa.

3.14 Example

Consider $X_1=\{u,v\}$ and $\tau= \{0, O_1, O_2, O_3, O_4, O_5, O_6, O_7, 1\}$ Where, $O_1=\{(u,0.6), (v,1)\}$, $O_2=\{u,1), (v,0.6)\}$, $O_3=\{(u,0.7),(v, 0)\}$,$O_4=\{(u,0),(v,0.3)\}$,$O_5=\{(u,1), (v,0)\}$, $O_6=\{(u,0.6), (v,0.6)\}$, $O_7=\{(u,0.6),(v,0)\}$.

A fts X_1 is rgfb2 N but not rgfb$_3$ N. For $\lambda_1 =\{(u,0.4),(v,0.4)\}$, $\lambda_2=\{(u,0.3),(v,1)\}$.

None of the rgfb-OS μ_1 and μ_2 containing λ_1 and λ_2, such that $\mu_1 \bar{q} \mu_2$

3.15 Example

Consider $X_1=\{u,v\}$ and $\tau = \{0, O_1, O_2, O_3, O_4, O_5, O_6, 1\}$.

Where, $O_1=\{(u,0), (v,0.6)\}$, $O_2=\{(u,0.3), (v,0)\}$, $O_3=\{(u,1), (v,0.2)\}$, $O_4=\{(u,0.7), (v,1)\}$, $O_5=\{(u,0.7),(v,0.2)\}$,$O_6=\{(u,1), (v,0.6)\}$.

A fts X_1 is rgfb$_3$N but not rgfb$_2$N. For $\lambda_1 =\{(u,0.3), (v,0)\}$, $\lambda_2 =\{(u,0), (v, 0.8)\}$.

None of the rgfb-OS μ_1 and μ_2 contains λ_1 and λ_2, where $\mu_1 \wedge \mu_2$=0.

3.16 Proposition

In fts X_1, every rgfb$_3$ N is rgfb$_4$ N.

Proof. Obvious, for $\lambda_1 \bar{q} \lambda_2$, $\lambda_1 \wedge \lambda_2$= 0 holds good. Where, λ_1 & λ_2 are fCSs.

The opposite of above statement is incorrect. This is shown as follows –

3.17 Example

Consider $X_1=\{u,v\}$ and $\tau= \{0, O_1, O_2, O_3, O_4, O_5, O_6, 1\}$.

Where, $O_1=\{(u,0), (v,0.6)\}$, $O_2=\{(u,0.3), (v,0)\}$, $O_3=\{(u,1), (v,0.2)\}$, $O_4=\{(u,0.7), (v,1)\}$, $O_5=\{(u,0.7),(v,0.2)\}$,$O_6=\{(u,1), (v,0.6)\}$.

A fts X_1 is rgfb$_4$N but not rgfb$_3$N. For $\lambda_1=\{(u,0.7),(v,1)\}$, $\lambda_2=(u,0.3),(v,0.8)\}$.

None of the rgfb-OS μ_1 and μ_2 containing λ_1 and λ_2, such that $\mu_1 \bar{q} \mu_2$

3.18 Proposition

In fts X_1, every rgfb$_2$ N is rgfb$_4$ N.

Proof. Obvious, for rgfb-OS μ_1 and μ_2, $\mu_1 \wedge \mu_2 = 0$, $\mu_1 \bar{q} \mu_2$ exists. The opposite of above statement is incorrect. This is shown as follows –

3.19 Example

Consider $X_1=\{u,v\}$ and $\tau = \{0, O_1, O_2, O_3, O_4, O_5, O_6, 1\}$. Where $O_1=\{(u,0), (v,0.6)\}$, $O_2=\{(u,0.3), (v,0)\}$, $O_3=\{(u,1), (v,0.2)\}$, $O_4=\{(u,0.7), (v,1)\}$, $O_5=\{(u,0.7), (v,0.2)\}$, $O_6=\{(u,1), (v,0.6)\}$. A fts X_1 is rgfb$_4$ N but not rgfb$_2$N. For $\lambda_1 =\{(u,1), (v,0.4)\}$, $\lambda_2 =\{(u,0),(v,0.4)\}$.

None of the rgfb-OS μ_1 and μ_2 contains λ_1 and λ_2, where $\mu_1 \wedge \mu_2 = 0$

3.20 Proposition

In fts X_1, every rgfb$_1$ N is rgfb$_4$ N.

Proof. Obvious, for fCSs λ_1& λ_2, $\lambda_1 \bar{q} \lambda_2$ the result $\lambda_1 \wedge \lambda_2 = 0$ and for $\mu_1 \wedge \mu_2 = 0$. The result exists. Where μ_1 and μ_2 are rgfb-OSs The opposite of above statement is incorrect. This is shown as follows –

3.21 Example

Consider $X_1=\{u,v\}$ and $\tau = \{0, O_1, O_2, O_3, O_4, O_5, O_6, 1\}$ Where, $O_1=\{(u,0), (v,0.6)\}$, $O_2=\{(u,0.3), (v,0)\}$, $O_3=\{(u,1), (v,0.2)\}$, $O_4=\{(u,0.7), (v,1)\}$, $O_5=\{(u,0.7), (v,0.2)\}$, $O_6=\{(u,1), (v,0.6)\}$. A fts X_1 is rgfb$_4$ N but not rgfb$_1$N. For $\lambda_1=\{(u,1), (v,0.4)\}$, $\lambda_2=\{(u,0.3), (v,0)\}$.

None of the rgfb-OS μ_1 and μ_2 contains λ_1 and λ_2, where $\mu_1 \wedge \mu_2 = 0$

3.22 Remark

From the above definition and example one can notice that the following Interrelations amoung rgfb normal spaces in fts.

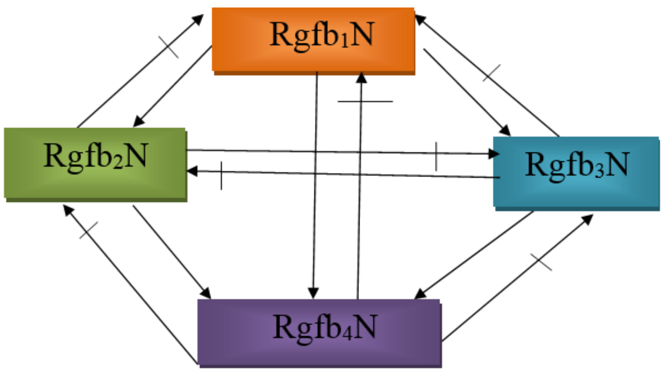

Fig3: Relationships among rgfb normal spaces in fts

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